**Multiaccess Communication**

- Satellite systems, radio networks (WLAN), Ethernet segment
- The received signal is the sum of attenuated transmitted signals from a set of other nodes, corrupted by distortion, delay and noise
- The multiaccess medium is allocated among the various nodes by the MAC (medium access control) sublayer
- We can view multiaccess communication in queueing terms: Each node has a queue of packets to be transmitted and the multiaccess channel is a common server.
Multiaccess Communication

Ideally, the server should view all the waiting packets as one combined queue to be served by an appropriate queueing discipline.

Problem: The server doesn’t know which nodes contain packets.

Problem: The nodes are unaware of packets at other nodes.

Knowledge about the state of the queue is distributed.
Slotted Multiaccess Model

- Slotted system: Same length of all packets, each packet needs one time unit (a slot) for transmission. All transmitters are synchronized, transmission starts at an integer time and ends before next integer time.

- Poisson arrivals: Packets arrive at each of the $m$ transmitting nodes according to independent Poisson processes. Let $\lambda$ be the overall arrival rate, and $\lambda/m$ the arrival rate at each transmitting node.

- Collision or perfect reception: If two or more nodes sends in a time slot there is a collision and the receiver obtains no information about the content or sources of the transmitted packages. If just one node sends the packet is correctly received.
Slotted Multiaccess Model (cont.)

- $(0, 1, e)$ Immediate Feedback: At the end of each slot, each node obtains feedback specifying whether 0 packets (idle), 1 packet (successful transmission), or more that 1 packet (collision/error) were transmitted in that slot.

- Retransmission of collisions: Each packet involved in a collision will be retransmitted in some later slot, until successfully transmitted. A node with a packet that must be retransmitted is said to be backlogged.
Slotted Multiaccess Model (cont.)

One of the two following is assumed

(a) No buffering: If one packet at a node is currently waiting for transmission or colliding with another packet during transmission, new arrivals at that node are discarded and never transmitted.

(b) Infinite set of nodes ($m = \infty$): The system has an infinite set of nodes and each newly arriving packet arrives at a new node.
Slotted Multiaccess Model (cont.)

With large number of nodes, relatively small arrival rate $\lambda$, and small required delay, new arrivals at backlogged nodes are almost negligible. Thus in this case the delay with assumption (a) should be relatively close to one with buffering. For a wide variety of systems with buffering and flow control assumption (a) will give a lower bound to the delay.

The assumption (b) provides an upper bound for the delay. Each of a finite set of nodes can consider itself as a set of virtual nodes and apply the given algorithm independently for each arrived packet. In this approach a node with several backlogged packets will sometimes cause a sure collision, by avoiding that we can get smaller delays.
Slotted Aloha

The Aloha network was developed around 1970 to provide radio communication between central computer and various data terminals at the campuses of University of Hawaii.

The basic idea is an unbacklogged node transmits a newly arriving packet in the first slot after packet arrival, thus risking occasional collisions but achieving very small delay if collisions are rare.

When a collision occurs each node sending one of the colliding packets discovers the collision at the end of the slot and becomes backlogged.

Backlogged nodes wait for some random number of slots before retransmitting.
Slotted Aloha, preliminary analysis

- Using infinite node assumption (b)
- New arrivals transmitted in a slot is a Poisson random variable with rate $\lambda$
- If retransmissions from backlogged nodes are sufficiently randomized, it is plausible to approximate the total number of retransmissions and new transmissions in a slot as a Poisson random variable with rate $G > \lambda$
- The probability that $k$ nodes will transmit in a slot is then

$$\frac{G^k}{k!} e^{-G}$$
Slotted Aloha, preliminary analysis

- The probability of a successful transmission is thus $Ge^{-G}$.

- In equilibrium the arrival rate, $\lambda$, should be equal to the departure rate, $Ge^{-G}$

- Maximum possible departure rate occurs at $G = 1$ and is $1/e \approx 0.368$

- For arrival rate $\lambda < 1/e$ there are two values of $G$ for which arrival rate equals departure rate!

- We ignore the dynamics of the system, as number of backlogged packets changes the parameter $G$ will change.
Slotted Aloha, analysis

Assume that each backlogged node retransmits with some fixed probability $q_r$ in each successive slot until a successful transmission occurs.

Number of slots from a collision until a given node involved in the collision retransmits is a geometric random variable, i.e. the probability that the number of slots is $i \geq 1$ is $q_r(1 - q_r)^{i-1}$.

We will now first assume the no-buffering assumption (a).

The behaviour is now described by a discrete time Markov model where the state is the number $n$ of backlogged nodes.
Slotted Aloha, analysis

- Each of the $n$ backlogged nodes will transmit, independently of each other, with probability $q_r$

- Each of the $m - n$ other nodes will transmit if one (or more) packets arrived during the previous slot

- Since arrivals are Poisson distributed with rate $\lambda/m$, the probability of no arrivals is $e^{-\lambda/m}$; thus the probability that an unbacklogged node will transmit is $q_a = 1 - e^{-\lambda/m}$

- Let $Q_a(i, n)$ be the probability that $i$ unbacklogged nodes transmit when $n$ nodes are backlogged

- Let $Q_r(i, n)$ be the probability that $i$ backlogged nodes transmit when $n$ nodes are backlogged
Slotted Aloha, analysis

We get

\[ Q_a(i, n) = \binom{m-n}{i} (1 - q_a)^{m-n-i} q_a^i \]

\[ Q_r(i, n) = \binom{n}{i} (1 - q_r)^{n-i} q_r^i \]

- The state \( n \) (number of backlogged nodes) increases by the number of new arrivals if we get a collision.
- The state decreases by 1 if we get no new arrivals and a successful transmission (of a retransmitted packet).
- The state is unchanged if we get a new arrival that is successfully transmitted, or if no retransmission occurs, or if retransmission occurs with collision.
If we get \( i \geq 2 \) new arrivals, collision will occur and the state will increase with \( i \), thus the transition probability
\[
P_{n,n+i} = Q_a(i, n)
\] in this case (\( 2 \leq i \leq m - n \)).

If we get one new arrival and a retransmission, collision will occur and the state will increase with 1,
\[
P_{n,n+1} = Q_a(1, n)(1 - Q_r(0, n))
\]

If we get one new arrival and no retransmission, or no new arrival and no or at least two retransmission, the state will be unchanged
\[
P_{n,n} = Q_a(1, n)Q_r(0, n) + Q_a(0, n)(1 - Q_r(1, n))
\]

If we get no new arrivals and one (i.e. successful) retransmission the state will decrease by 1,
\[
P_{n,n-1} = Q_a(0, n)Q_r(1, n)
\]
Slotted Aloha, analysis

The steady-state probabilities $p_n$ will satisfy

$$p_n = \sum_{i=0}^{n+1} p_i P_{i,n}, \quad \sum_{n=0}^{m} p_n = 1$$

This gives

$$p_{n+1} = \frac{1}{P_{n+1,n}} \left( p_n (1 - P_{n,n}) - \sum_{i=0}^{n-1} p_i P_{i,n} \right)$$

With this we can recursively express all $p_n$ in $p_0$ and then use $\sum_{n=0}^{m} p_n = 1$ to find the value of $p_0$ and thus all $p_n$. 

Slotted Aloha, analysis

Now we can compute the expected number of backlogged nodes as

\[ N = \sum_{n=0}^{m} np_n \]

With Little’s theorem we get the average delay \( T = N/\lambda \)
Little’s theorem

- $N(t)$ is number of packets waiting in a queue (here nr of backlogged packets) at time $t$
- $\alpha(t)$ is the number of packets that arrived in the interval $(0, t)$
- $T_i$ is the time spent in the queue by packet $i$,
- The time average of $N(t)$ up to $t$ is

$$N_t = \frac{1}{t} \int_{0}^{t} N(\tau) \, d\tau$$

- Normally $N_t$ changes with $t$, but many systems has a steady state as $t$ increases, i.e. $N = \lim_{t \to \infty} N_t$ exists.
Little’s theorem

- We also have the time average arrival rate over the interval \((0, t)\)

\[
\lambda_t = \frac{\alpha(t)}{t}
\]

- And the steady-state arrival rate \(\lambda = \lim_{t \to \infty} \lambda_t\) (assuming it exists)

- The time average of packet delay up to time \(t\) is

\[
T_t = \frac{\sum_{i=0}^{\alpha(t)} T_i}{\alpha(t)}
\]

- And the steady-state time average packet delay \(T = \lim_{t \to \infty} T_t\) (assuming it exists)
Little’s theorem

- If the steady state averages $N$, $\lambda$ and $T$ exists, Little’s theorem says that $N = \lambda T$

- The proof is to look at the area between the arrival function $\alpha(t)$ and the departure function $\beta(t)$ in two ways.

- This yields

$$\int_{0}^{t} N(\tau) \, d\tau = \sum_{i=0}^{\alpha(t)} T_i$$

- Dividing by $t$ yields

$$N_t = \frac{1}{t} \sum_{i=0}^{\alpha(t)} T_i = \frac{\alpha(t)}{t} \sum_{i=0}^{\alpha(t)} T_i = \lambda_t T_t$$
Slotted Aloha, instability

- This steady state analysis doesn’t tell the whole truth!
- We want retransmission probability $q_r$ to be relatively large to avoid large delays after collisions.
- If arrival rate is small, and few packets are involved in collisions, this works well and retransmissions are normally successful.
- But, if we get enough backlogged packets $n$ such that $q_r n \gg 1$, we get collisions in almost all successive slots and the system remains heavily backlogged for a long time.
We define the drift, $D_n$, in state $n$ as the expected change in backlog over one slot time, starting in state $n$.

$D_n$ is the expected number of new arrivals accepted into the system $(m - n)q_a$ less the expected number of successful transmissions in the slot, i.e. probability of successful transmission, $P_s$.

Thus $D_n = (m - n)q_a - P_s$ where

$$P_s = Q_a(1, n)Q_r(0, n) + Q_a(0, n)Q_r(1, n)$$

The attempt rate $G(n)$ is the expected number of attempted transmission in a slot when the system is in state $n$. 
Slotted Aloha, instability

- We have \( G(n) = (m - n)q_a + nq_r \)
- By inserting the definition of \( Q_a \) and \( Q_r \) in \( P_s \) we get

\[
P_s = (m - n)(1 - q_a)^{m-n-1}q_a(1 - q_r)^n + 
+ (1 - q_a)^{m-n}n(1 - q_r)^{n-1}q_r 
= \left( (m - n)q_a \frac{1}{1 - q_a} + nq_r \frac{1}{1 - q_r} \right) (1 - q_a)^{m-n}(1 - q_r)^n 
\approx ((m - n)q_a + nq_r) e^{-q_a(m-n)-q_r n} = G(n)e^{-G(n)}
\]

where the approximation is good if \( q_a \) and \( q_r \) are small
- The probability of an idle slot is approximately \( e^{-G(n)} \)
Slotted Aloha, instability

Thus the probability of number of packets in a slot is well approximated by a Poisson process random variable with rate $G(n)$, note that the rate varies with the state.

By plotting $P_s = G(n)e^{-G(n)}$ and the line $(m - n)q_a$ (as function of $n$) we can see the drift $D_n$ as the difference between the curve and the line.

Since the drift is the expected change in state from one slot to the next, the system tends to move in the direction of the drift although it may fluctuate.